Guaranteed Characterization of the Area Explored by an Autonomous Robot

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1. Introduction
2. Problem Statement
3. Problem Approach
4. Calculating the Winding Number
5. Daurade Mission
6. Conclusions and Future Work

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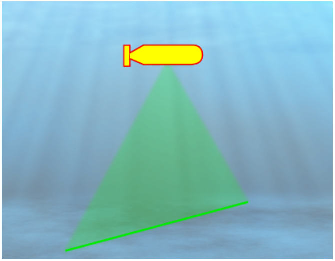
Introduction

Explored Area

The explored area is the union of the visible areas over the whole trajectory.



Introduction

Explored Area

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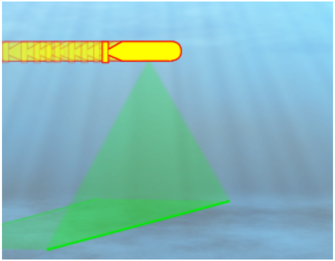
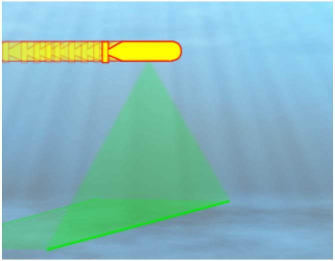


Problem

- Calculate the explored area.
- Calculate the number of times each point in the environment has been explored.



Applications

- Assess area-covering missions
  - Determine if a mission is complete
  - Plan other missions to fill possible gaps
- **Guarantee** that if a target is not detected, the target does not exist.

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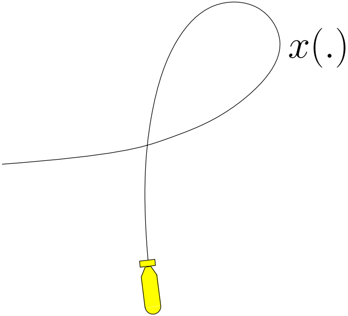
Problem Statement

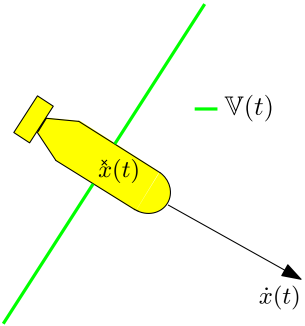
Hypothesis

- $x(.) : \mathbb{R} \rightarrow \mathbb{R}^2$
- $T = [0, T_{max}]$
- $x(.)$ is continuous in T.
- $x(.)$ and $\dot{x}(.)$ are known.



Problem Statement

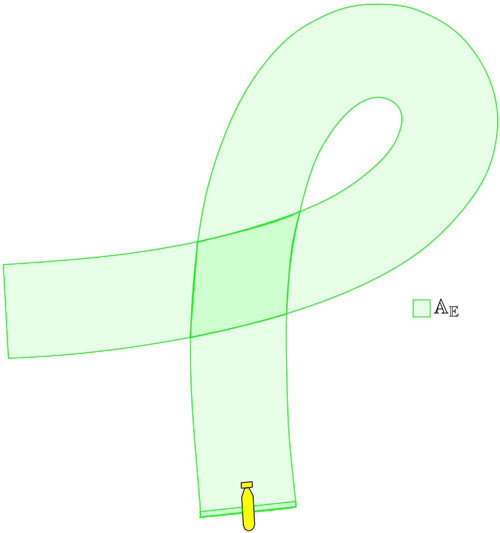
- $\mathbb{V}(x(t), \dot{x}(t))$ is the visible area at time t.



Problem Statement

$\mathbb{A}_{\mathbb{E}}$ is the explored area

$$\mathbb{A}_{\mathbb{E}} = \bigcup_{t \in T} \mathbb{V}(t)$$



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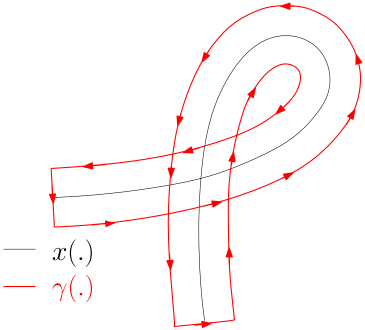
Sonar Contour

From the robot's trajectory $x(.)$ and the knowledge of the range of visibility of each observation sensor, the sonar contour $\gamma(.)$ can be defined as illustrated.

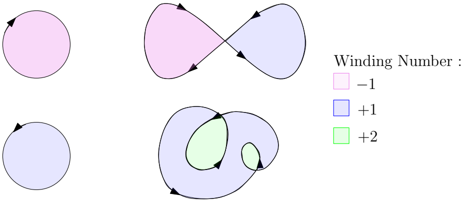
- $\gamma(.) : \mathbb{R} \to \mathbb{R}^2$
- $T_\gamma = [0, 1]$
- $\gamma(.)$ is continuous in T_γ.
- $\gamma(0) = \gamma(1)$.



Winding Number

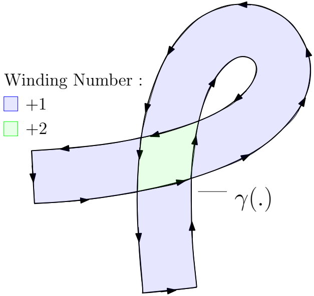
Winding Number

The winding number $\eta(\gamma(.), p)$ of a closed curve $\gamma(.)$ in the plane around a given point p is an integer representing the total number of times that curve travels counterclockwise around the point. [1]



Problem Approach

$$\mathbb{A}_{\mathbb{E}} = \{z \in \mathbb{R}^2 | \eta(\gamma(.), z) \neq 0\}$$



Problem Approach

Entries

- $\gamma(.)$, the sonar's contour.
- $\dot{\gamma}(.)$, the contour's derivative.

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- $\gamma(.)$, the sonar's contour.
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Desired Output

- Guaranteed approximation of the explored area $\mathbb{A}_{\mathbb{E}}$.
- Number of times each point in $\mathbb{A}_{\mathbb{E}}$ was seen during the mission.

Proposed solution

$$\eta(\gamma(.), .)$$

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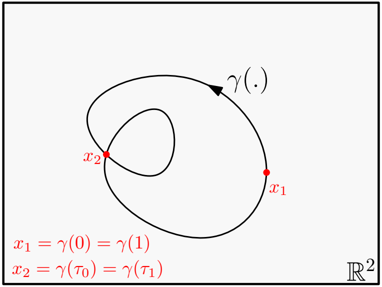
Self Intersections

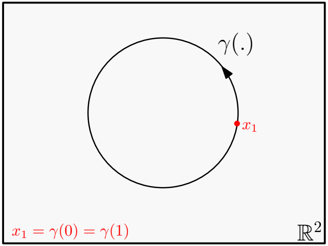
- Let $\gamma(.)$ be a continuous cycle, $\gamma(.) : [0,1] \rightarrow \mathbb{R}^2$,
- A self-intersection of $\gamma(.)$ determined by parameters (τ_0, τ_1), is a point x such that $x = \gamma(\tau_0) = \gamma(\tau_1)$, with $0 \leq \tau_0 < \tau_1 \leq 1$.



Simple Cycle

Simple Cycle (Jordan Curve)

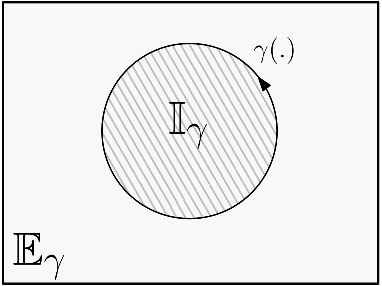
- $\gamma(.)$ is a simple cycle if it has only one self-intersection determined by the unique pair $(0, 1)$.
- $\gamma(.)$ is homeomorphic to S^1.

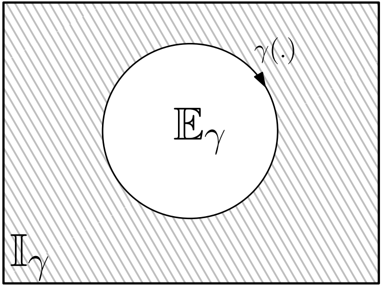


Simple Cycle

Jordan Curve Theorem

If $\gamma(.)$ is a simple cycle, then it divides the plane into two regions, $\mathbb{I}_\gamma$ and $\mathbb{E}_\gamma$.





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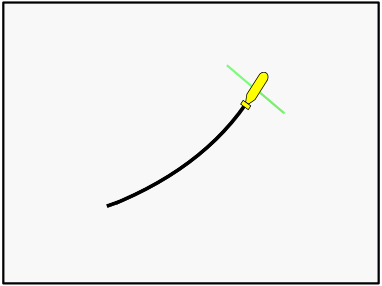

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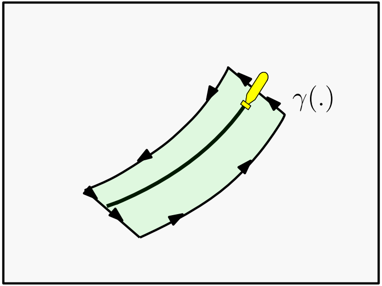

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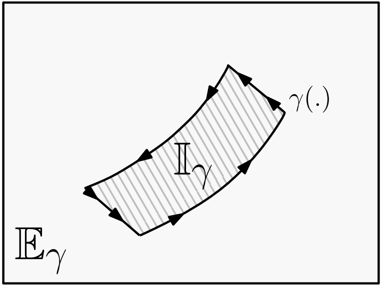

The winding number of simple cycles

If $\gamma(.)$ is a simple cycle and $p \in \mathbb{R}^2 \backslash \gamma(.)$,

$$\eta(\gamma(.), p) = \begin{cases} \chi_{\mathbb{I}_\gamma}(p) - 1 & \text{, if } \gamma(.) \text{ is clockwise oriented} \\ \chi_{\mathbb{I}_\gamma}(p) & \text{otherwise} \end{cases}$$

Where, $\chi_{\mathbb{I}_\gamma}(.)$ is the characteristic function of γ's interior set $\mathbb{I}_\gamma$.

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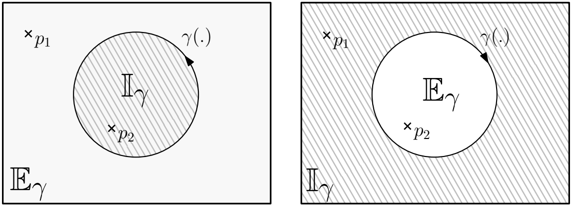


$\eta(\gamma(.), p_1) = 0$

$\eta(\gamma(.), p_2) = 1$

$\eta(\gamma(.), p_1) = 1 - 1 = 0$

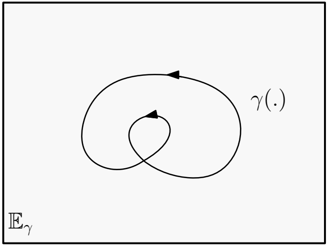
$\eta(\gamma(.), p_2) = 0 - 1 = -1$

The winding number of non-simple cycles

Hypothesis

$\gamma(.)$ is a continuous non-simple cycle:

- $\gamma(.) : [0,1] \to \mathbb{R}^2$,
- $\gamma(0) = \gamma(1)$,
- $\exists(\tau_0, \tau_1), 0 < \tau_0 < \tau_1 < 1 | \gamma(\tau_0) = \gamma(\tau_1)$.



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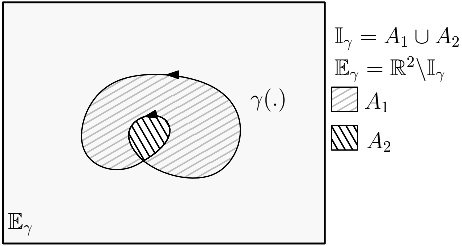
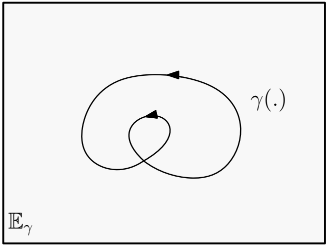


The winding number of non-simple cycles

Cell Decomposition :



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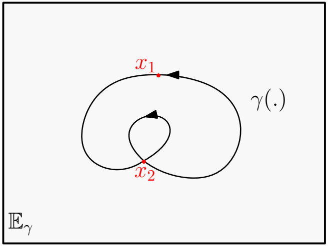

- **Self-intersections:**
  $X_\gamma = \{x \in \mathbb{R}^2 |\ \exists(\tau_0, \tau_1), 0 \leq \tau_0 < \tau_1 \leq 1, x = \gamma(\tau_0) = \gamma(\tau_1)\}$,

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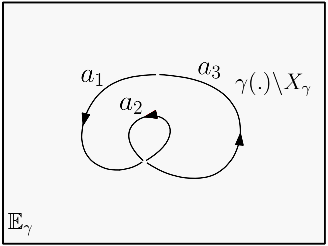

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- **Edges :** Connected components of $\gamma(.)\backslash X_\gamma$,

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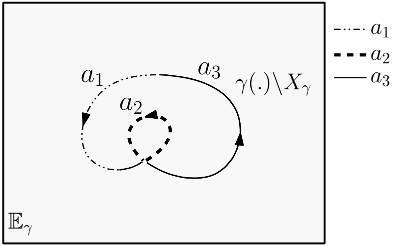

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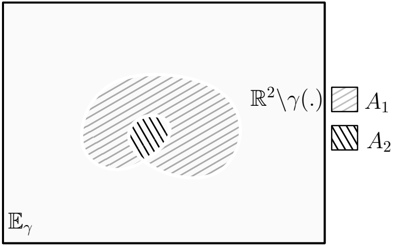

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- **Edges :** Connected components of $\gamma(.) \backslash X_\gamma$,
- **Cells :** Connected components of $\mathbb{R}^2 \backslash \gamma(.)$.

The winding number of non-simple cycles

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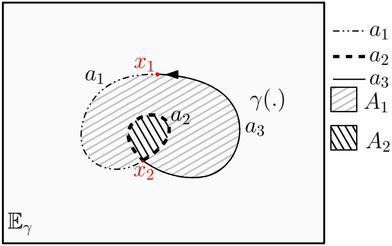

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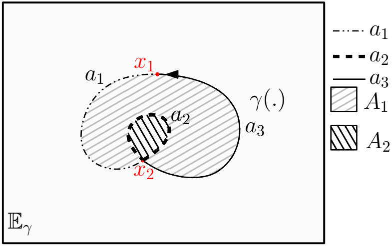
- **Edges :** Connected components of $\gamma(.) \backslash X_\gamma$,
- **Cells :** Connected components of $\mathbb{R}^2 \backslash \gamma(.)$.

The winding number of non-simple cycles

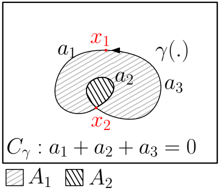
- A cycle can be algebraically described as a sum of all its edges, this sum results in zero.
- A cycle is simple if no other cycle can be written as a boolean combination of its edges.



$$C_\gamma : a_1 + a_2 + a_3 = 0$$

$$C_{x_2} : a_2 = 0$$

The winding number of non-simple cycles

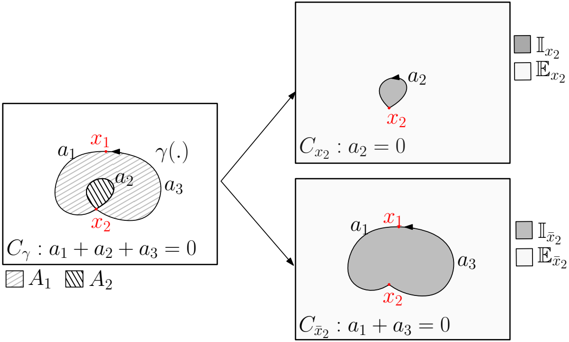


$$C_\gamma : a_1 + a_2 + a_3 = 0$$

$$C_{x_2} : a_2 = 0 \ , \ \mathbb{I}_{x_2} = A_2$$

$$C_{\bar{x}_2} : a_1 + a_3 = 0 \ , \ \mathbb{I}_{\bar{x}_2} = A_1 + A_2$$

The winding number of non-simple cycles



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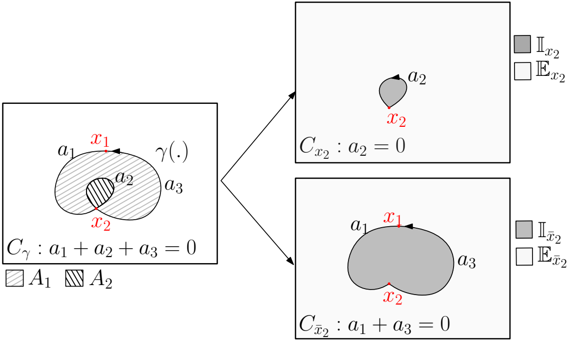
$$C_{\bar{x}_2} : a_1 + a_3 = 0 \ , \mathbb{I}_{\bar{x}_2} = A_1 + A_2$$

$$C_\gamma = C_{x_2} + C_{\bar{x}_2} = 0$$

$$\mathbb{I}_\gamma = A_1 + 2A_2$$

The winding number of non-simple cycles



$$C_\gamma = C_{x_2} + C_{\bar{x}_2} = 0 \ , \ \mathbb{I}_\gamma = A_1 + 2A_2$$

For $p \in \mathbb{R}^2 \backslash \gamma(.)$,

$$\eta(\gamma(.), p) = \chi_{\mathbb{I}_{x_2}}(p) + \chi_{\mathbb{I}_{\bar{x}_2}}(p)$$

The winding number of non-simple cycles

Let $\gamma(.)$ be a continuous cycle and $p \in \mathbb{R}^2 \backslash \gamma(.)$,

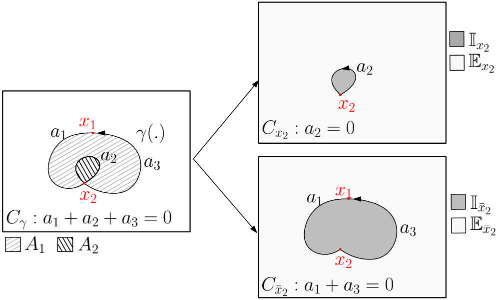
$$\eta(\gamma(.), p) = -Z_\gamma + \sum_{u \in U_\gamma} \chi_u(p)$$

Where,

- U_γ is the generating set of simple cycles such that $\sum_{u \in U_\gamma} u = C_\gamma$,
- Z_γ is the number of clockwise oriented cycles in U_γ,
- $\chi_{\mathbb{I}_u}(.)$ is the characteristic function of u's interior set $\mathbb{I}_u$.

The winding number of non-simple cycles

Example:



$$U_\gamma = \{C_{x_2}(.), C_{\bar{x}_2}\} \text{ and } Z_\gamma = 0$$

For $p \in \mathbb{R}^2 \setminus \gamma(.)$,

$$\eta(\gamma(.), p) = -Z_\gamma + \sum_{u \in U_\gamma} \chi_u(p) = \chi_{\mathbb{I}_{x_2}}(p) + \chi_{\mathbb{I}_{\bar{x}_2}}(p)$$

The winding number of non-simple cycles

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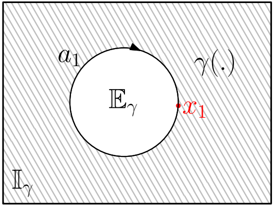


$C_\gamma : a_1 = 0$

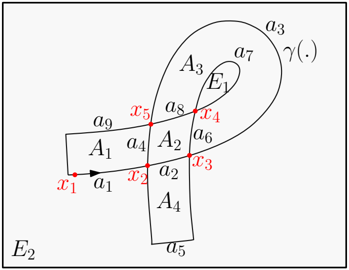
$$U_\gamma = \{C_\gamma\} \text{ and } Z_\gamma = 1$$

For $p \in \mathbb{R}^2 \backslash \gamma(.)$,

$$\eta(\gamma(.), p) = -Z_\gamma + \sum_{u \in U_\gamma} \chi_u(p) = \chi_{\mathbb{I}_\gamma} - 1$$

The winding number of non-simple cycles

Example:



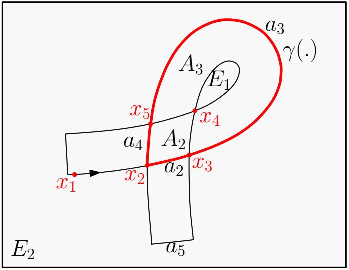
$C_\gamma : a1 + a_2 + a_3 + a_4 + a_5 + a_6 + a_7 + a_8 + a_9 = 0$

Cell Decomposition :

- $X_\gamma = \{x_1, x_2, x_3, x_4, x_5\}$
- $a_\gamma = \{a_1, a_2, a_3, a_4, ..., a_9\}$
- $A_\gamma = \{A_1, A_2, A_3, A_4, E_1, E_2\}$

The winding number of non-simple cycles

Example:



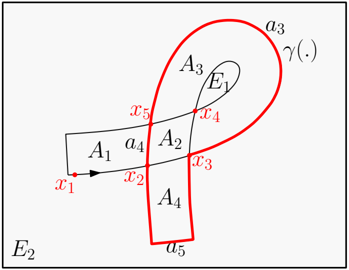
$$C_\gamma : a1 + a_2 + a_3 + a_4 + a_5 + a_6 + a_7 + a_8 + a_9 = 0$$

$$C_{x_2} : a_2 + a_3 + a_4 = 0$$

$$\mathbb{I}_{x_2} = A_2 + A_3 + E_1$$

The winding number of non-simple cycles

Example:



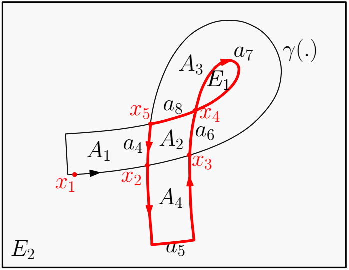
$C_\gamma : a1 + a_2 + a_3 + a_4 + a_5 + a_6 + a_7 + a_8 + a_9 = 0$

$C_{x_3} : a_3 + a_4 + a_5 = 0$

$\mathbb{I}_{x_3} = A_2 + A_4 + A_3 + E_1$

The winding number of non-simple cycles

Example:



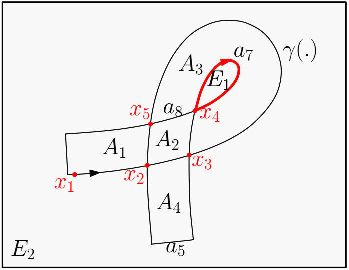
$$C_\gamma : a1 + a_2 + a_3 + a_4 + a_5 + a_6 + a_7 + a_8 + a_9 = 0$$

$$C_{x_5} : a_4 + a_5 + a_6 + a_7 + a_8 = 0$$

$$\mathbb{I}_{x_5} = A_2 + A_4 - E_1$$

The winding number of non-simple cycles

Example:



$$C_\gamma : a1 + a_2 + a_3 + a_4 + a_5 + a_6 + a_7 + a_8 + a_9 = 0$$

$$C_{x_5} : a_4 + a_5 + a_6 + a_7 + a_8 = 0$$

$$C_{x_4} : a_7 = 0$$

$$\mathbb{I}_{x_4} = -E_1$$

Decomposition by elimination

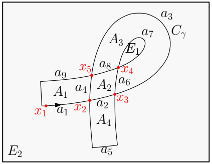
Algorithm

$C_r = C_\gamma$,

1. Choose a simple cycle c in C_r,
2. Add c to U_γ,
3. Remove c from C_r: $C_r - c$,
4. If C_r is not simple, go to 1.

Add C_r to U_γ.

Example:



$$C_r : a_1 + a_2 + a_3 + a_4 + a_5 + a_6 + a_7 + a_8 + a_9 = 0,$$

Decomposition by elimination

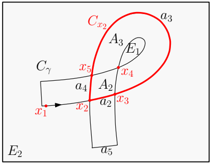
Algorithm

$C_r = C_\gamma$,

1. Choose a simple cycle c in C_r,
2. Add c to U_γ,
3. Remove c from C_r: $C_r - c$,
4. If C_r is not empty, go to 1.

Add C_r to U_γ.

Example:



$C_r : a_1 + a_2 + a_3 + a_4 + a_5 + a_6 + a_7 + a_8 + a_9 = 0$,

1. $c = C_{x_2} : a_2 + a_3 + a_4 = 0$
2. $U_\gamma = \{C_{x_2}\}$,
3. $C_r : C_r - c = a_1 + a_5 + a_6 + a_7 + a_8 + a_9 = 0$.

Decomposition by elimination

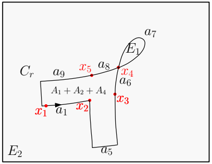
Algorithm

$C_r = C_\gamma$,

1. Choose a simple cycle c in C_r,
2. Add c to U_γ,
3. Remove c from C_r: $C_r - c$,
4. If C_r is not empty, go to 1.

Add C_r to U_γ.

Example:



$$C_r : a_1 + a_5 + a_6 + a_7 + a_8 + a_9 = 0,$$

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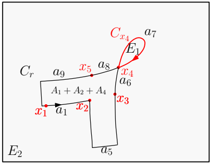

$C_r : a_1 + a_5 + a_6 + a_7 + a_8 + a_9 = 0,$

1. $c = C_{x_4} : a_7 = 0$
2. $U_\gamma = \{C_{x_2}, C_{x_4}\},$
3. $C_r : C_r - c = a_1 + a_5 + a_6 + a_8 + a_9 = 0.$

Decomposition by elimination

Algorithm

$C_r = C_\gamma$,

1. Choose a simple cycle c in C_r,
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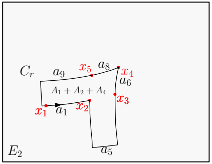
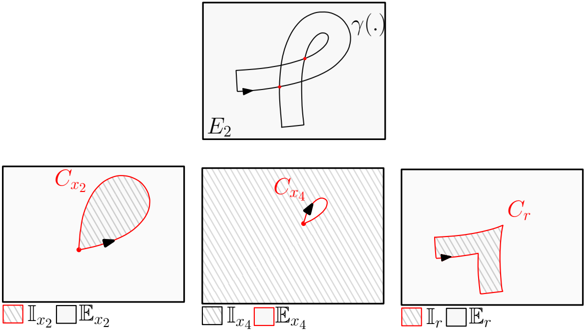


$C_r : a_1 + a_5 + a_6 + a_7 + a_8 + a_9 = 0$,

1. $c = C_{x_4} : a_7 = 0$
2. $U_\gamma = \{C_{x_2}, C_{x_4}\}$,
3. $C_r : C_r - c = a_1 + a_5 + a_6 + a_8 + a_9 = 0$.

$U_\gamma = \{C_{x_2}, C_{x_4}, C_r\}$,

The winding number of non-simple cycles



$$U_\gamma = \{C_{x_2}, C_{x_4}, C_r\} \text{ and } Z_\gamma = 1$$

$$\eta(\gamma(.), p) = -Z_\gamma + \sum_{u \in U_\gamma} \chi_u(p) = \chi_{\mathbb{I}_{x_2}} + \chi_{\mathbb{I}_{x_4}} + \chi_{\mathbb{I}_r} - 1$$

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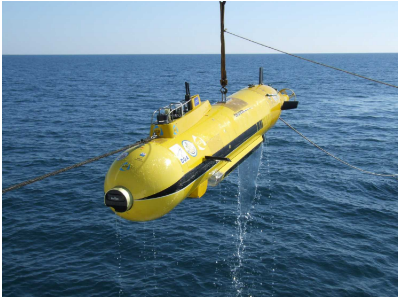
6 Conclusions and Future Work

Daurade



Courtesy of DGA/GESMA

Data

- DVL,
- IMU,
- Pressure.

Mission

- Classical survey path (law-mowing pattern),
- Roadstead of Brest (France, Brittany),
- 47 minutes.

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Conclusions

Using the topological properties of the robot's exteroceptive sensors contour, we are able to

- determine the are explored during a mission,
- determine the number of times a point in the space was in the robot's range of visibility,

Future Work

- Consider cases where the sensor's contour moves backwards,
- Test the algorithm in real time.

[1] S. G. Krantz.
The index or winding number of a curve about a point.
In *Handbook of Complex Variables*, pages 49–50, Boston, MA, 1999.